MnT Hints & Recap HW 2

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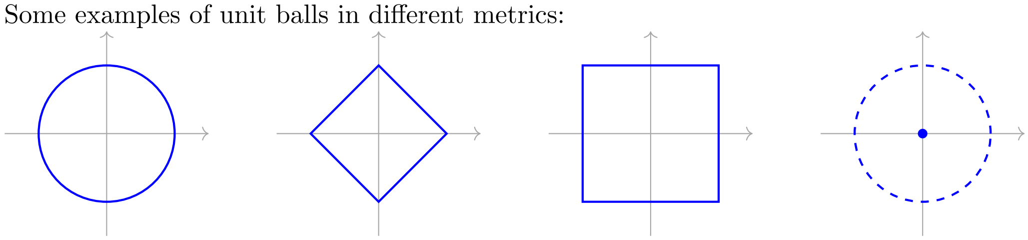
Recap

A metric space consists of a non-empty set X together with a function $d : X \times X \to \mathbb{R}$ such that the following hold:
1) $\forall\ x, y \in X,\ d(x, y) \geq 0;$ and $d(x, y) = 0 \iff x = y$
2) (Symmetry) $\forall\ x, y \in X,\ d(y, x) = d(x, y)$
3) (Triangle inequality) $\forall x, y, z \in X,\ d(x, z) \leq d(x, y) + d(y, z)$

Some examples of unit balls in different metrics:



Euclidean (d_2) Taxicab (d_1) Sup (d_∞) Discrete (d_0)

For the same set X, you can have several different d_X defined on it.

Definition 5.23 *A subset S of a metric space (X, d) is bounded if* $\exists\, x_0 \in X$ and $K \in \mathbb{R}$ such that $d(x,\, x_0) \leqslant K$ for all $x \in S$.

Definition 5.24 *If S is a non-empty bounded subset of a metric space with metric d, then the diameter of S is* $\sup\{d(x, y) : x, y \in S\}$. *The diameter of the empty set is 0.*

Proposition 5.26 Let (X, d) be a metric space. If $A_1, A_2, \ldots, A_n \subseteq X$ are bounded, then

$$\bigcup_{i=1}^{n} A_i \quad \text{is bounded;}$$

i.e., the union of any finite number of bounded subsets of a metric space is bounded.

Hints

Problem 5.1

There's an absolute value - can you break down the problem into two easier problems using what you know about the absolute value? Make sure to specify exactly what properties of metrics you are using at each step.

Problem 5.8

Read Definition 5.23 for one of the implications:)

Proving the other implication requires knowledge of open balls so feel free to attempt it after next lecture (at least that's the only way I know how to do it, maybe I'm wrong).

Problem 5.9

A fairly straightforward analysis of whether or not boundedness properties would also apply in A if they apply in B.

Problem 5.13

Proposition 5.41!! Literally just scroll up in the textbook it's right there lmao.

For the other implication, set up an 'equal to' statement so you can use the double inclusion lemma (note: this isn't compulsory, but helpful to get a complete solution) and then you can perhaps work backwards.

Problem 5.17

Read definition 5.3, problem 5.2, and relate d_1 to what is shown in 5.2.

Problem 5.18

This problem uses open balls as well, so I think it's best if you attempt it after the next lecture. Regardless, it's simple enough, think of all sorts of weird metrics and check whether this is always true.