MnT Hints & Recap HW 0

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This one isn't graded, but it's still a good idea to actually write the proofs out :) Part I (Ch. 2) is for Tuesday, Part II (Ch. 3) for Friday.

Recap

The one lemma you will probably use forty times this term. To prove $A = B$ for sets, prove $A \subseteq B$ and $B \subseteq A$, aka double inclusion 🔥

To prove $A \subseteq B$, take $x \in A$ and get to $x \in B$.

Notation.
$B \setminus A = \{x \in B : x \notin A\}$; if $A \subseteq B$ this is the complement of A in B.
$x \in \bigcup_{i\in I} A_i \iff \exists\, i_0 \in I$ with $x \in A_{i_0}$, and $x \in \bigcap_{i\in I} A_i \iff \forall\, i \in I,\ x \in A_i$.
De Morgan: $S \setminus \bigcup_I A_i = \bigcap_I (S \setminus A_i)$ and $S \setminus \bigcap_I A_i = \bigcup_I (S \setminus A_i)$.
$A \times B = \{(a, b) : a \in A,\ b \in B\}$, ordered pairs.

Definition 3.1. The (direct, forward) image of $A \subseteq X$ under $f : X \to Y$ is $f(A) = \{y \in Y : y = f(a) \text{ for some } a \in A\}$.
Definition 3.2. The inverse image of $C \subseteq Y$ is $f^{-1}(C) = \{x \in X : f(x) \in C\}$.
NB. $f^{-1}(C)$ makes sense for *every* map f. It does not need an inverse function to exist. The notation is kinda meh and many get confused by it at least once. Also $f^{-1}(y)$ means $f^{-1}(\{y\})$ and can have many elements, or none.

Draw a picture for Prop. 3.14!

Hints: Part I (due TUE 1.09)

Tip: '=' probably means double inclusion :)

Problem 2.1
Take x in the left-hand side, write down all three things you now know about it, then read

off the two you need. The only step with any content is noticing that $D \subseteq X$, so once you're coming back the other way you get $x \in X$ for free and don't need to justify it.

Problem 2.2
Same game.

Problem 2.3
The hypothesis $X \setminus V = X \cap U$ is the only thing you've been given, so every step should be spent either feeding something into it or reading something out of it. Concretely: if you know $x \in X$ and $x \notin U$, then $x \notin X \cap U$, so $x \notin X \setminus V \ldots$ and now what does that tell you, given $x \in X$?

Also use $V \subseteq X \subseteq Y$ freely, that's what the chain is there for.

Problem 2.4
Elements here are ordered pairs, so start your proof with "let $(a, b) \in \ldots$" and not "let $x \in \ldots$". Then it's coordinatewise and honestly a bit boring. Boring is fine.

Problem 2.5
If you did 2.4 properly, this is the same proof with more subscripts 🔥

Problem 2.6
This is the first one that's actually about indexing sets rather than about $\cap$ and $\cup$.

Remember $x \in \bigcup_{i \in I} B_{i1}$ means there *exists* some index with $x \in B_{i1}$. Give that index a name, i_0, immediately. Do the same for V and get j_0. Now you're holding a pair $(i_0, j_0) \in I \times J$, which is exactly what the union on the right is indexed by, so you're done.

The reverse inclusion is easier and shorter than you expect, don't overthink it.

Hints: Part II (due FRI 4.09)

Tip: always write down what is given and what you w.t.s. before you start.

Problem 3.1
This is Definitions 3.1 and 3.2 and nothing else, which is the point of the exercise. Start with "let $y \in f(A)$, so $y = f(a)$ for some $a \in A$" and go on.

Problem 3.2
Not a proof problem, a "do you actually know what f^{-1} means" problem. Sketch sin first, then read the answers off the picture.

Two traps. First, sin is periodic, so an inverse image will typically be an infinite union over $n \in \mathbb{Z}$, not a single interval. Second, on each hump the condition $\sin x \leq 1/2$ cuts out *two* pieces, not one, so $f^{-1}([0, 1/2])$ has twice as many intervals per period as $f^{-1}([0, 1])$. And $\arcsin(1/2) = \pi/6$ is the number you need for the endpoints.

The last one, $f^{-1}([-1, 1])$, is a freebie 👀

Problem 3.3
Unfold the definitions on both sides and you'll find they're the same sentence, "$g(f(x)) \in U$", written in two different ways. So the proof more or less writes itself.

Do still write both inclusions!

Problem 3.4
Substitute $y = 2x$ everywhere and the whole thing collapses into one-variable algebra.

$f([0, 1])$ wants a description, so describe it in words as well as in set notation (it's a line segment, say where it starts and ends). For the square, ask which of the two conditions $0 \leq x \leq 1$ and $0 \leq 2x \leq 1$ is the binding one. For D you're solving $x^2 + (2x)^2 \leq 1$, which is a one-line inequality.

Proposition 3.14
The proof is in the textbook :)

The part actually worth thinking about is why you only get $C \cap f(X)$ and not C: f cannot possibly hit the part of C lying outside its image, so that part has nowhere to come from.